

Charles-Jean de La Vallée Poussin

**COLLECTED WORKS**  
**OEUVRES SCIENTIFIQUES**

Volume I

**Biography and Number Theory**  
**Biographie et théorie des nombres**

*Edited by*  
*Edité par*

**Paul Butzer      Jean Mawhin**  
**Pasquale Vetro**



ACADÉMIE ROYALE  
DE BELGIQUE



CIRCOLO MATEMATICO  
DI PALERMO

**2000**

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*Ch. de la Vallée Poussin*

Hommage à Charles de La Vallée Poussin 78  
 CH. J. DE LA VALLÉE POUSSIN  
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## Foreword

The proof of the prime number theorem of 1896 is often regarded as Charles-Jean DE LA VALLÉE POUSSIN's (VP, for short) highest achievement in mathematics. Conjectured by Carl Friedrich GAUSS a century earlier, it was also established independently in the same year by Jacques HADAMARD<sup>1</sup>, the proof being similar in principle but different in the technique. Whereas VP presented the three parts of his *Mémoire* to the *Société Scientifique de Bruxelles* in its sessions of January and April, 1896, Hadamard communicated his results to the *Académie des Sciences de Paris* in June 1896; in fact while reading the proofs of his paper he received VP's first part.

The work for which VP is most widely known is often said to be his *Cours d'Analyse Infinitésimale* (Vol. I, 1903; Vol. II, 1906). Whereas the first volume went through 12 editions (the last 1959), the second volume through 9 editions (the last 1957), the second edition (of 1909) was revolutionary in the sense that it already contained an exposition of the Lebesgue integral. The edition of 1914 (of which only the first volume appeared) was especially popular on account of its fully modern treatment of total derivatives for functions of several variables and the new developments of the Lebesgue integral covered. In fact, this *Cours d'Analyse* seems to have been the standard text in the subject,

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<sup>1</sup>Born 1865 in Versailles, died 1963 in Paris, studied 1884-1888 at the Ecole Normale Supérieure, taught 1890-1893 at the Lyceum Buffon and received his doctorate 1892. After being maître de conférences in Bordeaux 1893-1897, and at the Sorbonne, he became Professor at the Collège de France in 1909 and at the Ecole Polytechnique in Paris in 1912. He retired in 1937 and spent the war years 1940-1947 at Columbia University.

Hadamard's name has been associated with Hadamard's gap theorem, the Hadamard three circle theorem, the Hadamard factorization theorem for integral functions, Hadamard's determinants. His work on the theory of propagation of waves (monograph of 1903, Paris, Hermann) and his *Lectures on Cauchy's problem in linear partial differential equations* (Cambridge, New Haven, 1922) are just as important. But his proof of the prime number theorem of 1896 made Hardy describe him as the "living legend" in mathematics (1944). Hadamard was the author of some 340 papers; he continued working until being over 90.

Member of the Académie des Sciences in Paris since 1912, he was elected an associate of the Académie royale de Belgique in 1922, and was a foreign member of many academies. P. Levy, S. Mandelbrojt, B. Malgrange, P. Malliavin, *La vie et l'oeuvre de Jacques Hadamard*, L'Enseignement Math., Genève, 1967;

S. Mandelbrojt, L. Schwartz, Jacques Hadamard (1865-1963), *Bull. Amer. Math. Soc.*, 71 (1965), 107-129; M.L. Cartwright, Jacques Hadamard, *J. London Math. Soc.*, 40 (1965), 722-748; V. Maz'ya, T. Shaposhnikova, Jacques Hadamard, A Universal Mathematician, *Amer. Math. Soc.*, 1998; Oeuvres de Jacques Hadamard, 4 Vol., C.N.R.S., Paris, 1968; DSB



also in top-level North American universities, at least up to the late forties, in competition with the popular Édouard GOURSAT's *Cours d'Analyse Mathématique*, Gauthier-Villars, 1st ed., 2 Vols., Paris 1902-1905; 2nd ed., 3 Vol., Paris 1910-1911-1915, 7th ed. 1949; (transl. into English by E.R. HEDRICK and O. DUNKEL, Ginn, Boston 1916).

One editor (MAWHIN) has the privilege to be, at the *Université Catholique de Louvain*, since 1970, the great-grandson of VP on the Chair of differential and integral Calculus. The only fit of passion he remembers from his PhD thesis advisor at the *Université de Liège* happened at the library in the *Institut d'Astrophysique* of Liège, when discovering the stealing of an early edition of VP's *Cours d'analyse infinitésimale*

As to Lebesgue integration, whereas especially the second edition (1928) of Henri Lebesgue's *Leçons sur l'intégration et la recherche des fonctions primitives*, (Gauthier-Villars, Paris 1904), is often regarded as his most important contribution to mathematics (more so than any of his articles), it seems that VP's monograph *Intégrales de Lebesgue. Fonctions d'ensemble. Classe de Baire* (Borel Tracts, 1916; 2<sup>nd</sup> strongly augmented ed. 1934, taught at the Collège de France in 1915-1916) was at least as popular. In any case, it was the monograph from which one of the editors (BUTZER) had the pleasure to learn Lebesgue integration at Loyola College (now Concordia University), Montreal in 1947. His teacher, a former doctoral student of Joseph L. WALSH at Harvard University, used it in class.

As to approximation theory, VP's book *Leçons sur l'approximation des fonctions d'une variable réelle* (1919, based on lectures given at the Sorbonne 1918) was the first one in the subject. Whereas approximation theory may be said to have set in with work of Pafnuty Lvovitch CHEBYSHEV<sup>2</sup> (1854) in Russia, and was a chief area of research in mathematics in Russia, at least until c.

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<sup>2</sup>(Čebyšev, Tschebyscheff), born 1821 in Okatovo, Kaluga region of Russia, died 1894, entered Moscow university in 1834 and received his candidacy of mathematics degree in 1841. His master's degree followed in 1846; in both cases his advisor was N.D. Brashman (1796-1866) to whom Chebyshev attributed great influence on his development. He moved to St. Petersburg University in 1846, obtaining the *venia legendi* there in 1847, his doctorate in 1849, an extraordinary professorship in 1850, and full professorship in 1860. He was nominated in turn junior, extraordinary and ordinary academician of the St. Petersburg Academy of Science in 1853, 1856 and 1859, respectively.

Chebyshev, who was regularly in Paris, at the latest by 1852, a friend of Liouville and Hermite, was the author of circa 80 publications, covering approximation theory, probability theory, number theory, theory of mechanisms, as well as many problems of analysis and practical mathematics. He was also proud to be a constructor of various mechanisms, including an arithmomètre (a calculating machine), and a bicycle for women. He was the creator in St. Petersburg of the largest prerevolutionary school of mathematics in Russia. Its most prominent members included A.N. Korkin, E.I. Zolotarev, C.A. Posse, A.A. and V.A. Markov, A.M. Lyapunov, V.A. Steklov, G.F. Voronoi and A.N. Kylov.

Two papers of Chebyshev, published in 1854 and 1859, marked the beginning of his 40-year research on approximation theory (or constructive function theory, as the Russians call it since coined so by S.N. Bernstein). Thematically it included the theory of orthogonal polynomials, interpolation, theory of moments, integration, approximate quadrature, and continued fractions. This work was inspired by his interest in the theory of mechanisms,



1965, VP's monograph was practically the only one of its type until the Russian books by N.I. ACHIESER (of 1947), I.P. NATANSON (of 1951) and A.F. TIMAN (of 1960) appeared as translations in the West, namely in 1953, 1955 and 1963, respectively. VP's monograph is still one of the most elegant, lucid and modern flavoured books (although written about 80 years ago) on the subject, as one editor also had the pleasure to observe (in 1950) when writing his dissertation at Toronto University.

Nevertheless, a complete (collected) publication of his works has never been undertaken to this day. In this respect one must observe that VP was the author of in fact eight books and some 115 research papers. Many of these papers are just as important and influential although perhaps not as spectacular as his work in number theory and as the two books mentioned. His profound contributions encompassed apart from number theory, measure and integration theory and approximation theory, also trigonometric and Fourier series, almost periodic functions, analytic and quasi-analytic functions, conformal representation, potential theory [with two volumes in the subject of 1937 and 1949], ordinary differential equations, as well as diverse topics, such as algebra, geometry, mechanics, physics, probability theory. As to sampling theory of signal analysis, a field which has become especially popular in mathematical circles since only ca. 1975, VP in his 90-page paper of 1908 already provided the full foundation for this theory in the important instance of duration-limited signals.

The object of these *Collected Works* is not only the reproduction of all of VP's research papers but the placement of the overwhelming bulk of these in the context of present-day research by means of commentaries, tracing both their genesis and impact, written by eminent contemporary scholars. There is a definite reason why especially the *Collected Works* of VP should be made available to the mathematical community. Half of his papers appeared in the *Annales de la Société Scientifique de Bruxelles*, above all his earlier papers, including his particularly famous papers on number theory. Whereas this Society may have played an important role in local scientific circles at the time<sup>3</sup>, the *Annales* never had a wide circulation and are probably in the possession of only a few scattered libraries in the world. Thus for many mathematicians it will be the first time that they can study VP's papers at first hand.

The papers are organized by coherence of subject matter rather than by

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in particular James Watt's parallelogram and steam engine, which he learned to know during his grand tour of France, England and Germany of 1852. In fact, Chebyshev can be regarded as the founder of approximation theory. P.J. Davis (1983) argues that Russian success in space travel during the 1950s can be attributed to mathematicians of the Russian scientific establishment, intellectual great-grandsons of Chebyshev.

P.L. Butzer, F. Jongmans : P.L. Chebyshev (1821–1894), *Jour. Approx. Theory*, 96 (1999), 111–138; Chebyshev's Complete Collected Works appeared in *Izdatel'stvo Akad. Nauk SSR*, Moscow-Leningrad 1946–1951, in 5 Volumes (some 2000 pp.); A. Markoff, N. Sonin, "Oeuvres de P.L. Tchebychef", 2 Vols., St. Petersburg 1899/1907 (Reprint Chelsea, New York 1952).

<sup>3</sup>See *The mathematical environment of Charles-Jean de La Vallée Poussin at the Université Catholique de Louvain* in this volume.



chronology, so that VP's intellectual evolution may stand out.

Four obituary addresses concerning VP are reproduced in Volume I, namely those by P. MONTEL<sup>4</sup>, G. LEMAÎTRE<sup>5</sup>, J.C. BURKILL and Jean FAVARD<sup>6</sup>. They either give a very professional account of the work of VP or a characteristic portrait of his life. Montel was a long-standing friend of VP, Lemaître his famous student. The Favard address appeared in the proceedings of the Oberwolfach conference on approximation of 1963 dedicated to VP. Added is also an as yet unpublished biography of VP written by his youngest son, Charles de La Vallée Poussin. The authors have also added their own brief account of the life and work of VP; it is in no sense complete but intended to be complementary to the five addresses cited. There also exist several further excellent descriptions of the mathematical work of VP, namely that of Gustave VERRIEST<sup>7</sup> of 1928, those

<sup>4</sup>Born 1876 in Nice, died 1975 in Paris, studied 1894-1897 at the Ecole Normale in Paris, then taught at a Lyceum. He received his doctorate 1907. He taught from 1911 to 1946 at the Ecole Polytechnique and at the Faculté des Sciences de Paris, and parallel as professor of the Ecole nationale Supérieure des Beaux-Arts, was Director of the Ecole Pratique des Hautes Etudes, President of the Palais de la Découverte, and member of the Académie des Sciences since 1937. He was elected an associate of the Académie royale de Belgique in 1946.

Montel's main research dealt with complex function theory and sequences of functions. He introduced the central concept of normal families of holomorphic functions; the extension to meromorphic functions was made by him in 1912. His monograph *Leçons sur les familles normales de fonctions analytiques et leurs applications*, Gauthier-Villars, Paris, 1927, is a landmark in the field. Montel has also contributed to the approximation of complex functions through polynomials, conformal mappings, "schlicht" functions and the localization of zeros of polynomials.

Paul Montel : Mathématicien Niçois (Foreword by J. Médecin, Maire de Nice); *Comptes Rendus Acad. Sci. Paris*, 280 (1978), Vie académique, 186-188.

<sup>5</sup>See *The mathematical environment of Charles-Jean de La Vallée Poussin at the Université Catholique de Louvain*, this volume.

<sup>6</sup>Born 1902 in Peyrat-la-Nonière (Creuse, France), died 1965, studied, 1921-1924 at the École Normale Supérieure and at the Faculté des Sciences in Paris, and completed his studies with the agrégation de mathématiques. After a brief time at a Lyceum, he went to Copenhagen to work with H. Bohr on almost periodic functions, writing a brilliant thesis in the field 1927. He was elected maître de conférences at the University of Grenoble, then Professor there in? He was made a prisoner of war in 1940; in the camp he created a Faculté des Sciences of which he was dean. There he wrote his highly cited paper "Sur les multiplicateurs d'interpolation" (*J. Math. Pures Appl.*, (9) 23 (1944), 219-247).

After the war he took up his duties at the École Polytechnique in Paris, being Professor of Analysis. Favard worked in various fields, approximation theory, geometry of convex bodies, almost periodic functions, moment problems, topology and theory of measure, functional analysis. His book "Leçons sur les fonctions presque périodiques", Gauthier-Villars, Paris, 1935, and his *Cours d'analyse de L'École Polytechnique*, 4 Vols., 1960-63, are well-known. The concept of saturation and the associated Favard classes he introduced into approximation theory made him extremely popular in the field. At least a hundred authors wrote some 500 papers in the subject. He was president of the Société mathématique de France and an Officer of the Légion d'Honneur.

G. Alexits, M. Zamansky, Jean Favard 1902-1965, In: *Abstract Spaces and Approximation* (Proc. Conference Oberwolfach, July 1969; Eds. P.L. Butzer and B.Sz.-Nagy), ISNM Vol. 10, Birkhäuser, Basel/Stuttgart 1969, pp. 19-24.

<sup>7</sup>See *The mathematical environment of Charles-Jean de La Vallée Poussin at the Université Catholique de Louvain*, this volume.

of Ferdinand SIMONART<sup>8</sup> of 1926, 1927 and 1946, and that of L.GODEAUX<sup>9</sup> of 1967. These are not reproduced here, due to their length.

The mathematical environment of VP at the *Université Catholique de Louvain* (his teachers and colleagues) is described in an article by J. MAWHIN, VP's connections with the *Académie royale de Belgique* are given in an article by P.L. BUTZER and J. MAWHIN, and finally his relations with the *Circolo Matematico di Palermo* and other Italian academies are analyzed by P. NASTASI.

The editors believe that it would be of interest to the readers that short biographies (with references) be attached (as footnotes, also in the commentaries by the experts), to those mathematicians who were somehow or other in contact with VP.

The editors wish the readers of the volumes to have as much pleasure in studying them as did the editors when preparing them.

P.L. Butzer (Aachen)

J. Mawhin (Louvain-la-Neuve)

P. Vetro (Palermo)

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<sup>8</sup>See *The mathematical environment of Charles-Jean de La Vallée Poussin at the Université Catholique de Louvain*, this volume.

<sup>9</sup>Born 11.10.1887 in Morlanwelz (Belgium); died 21.4.1975 in Liège. After one year at the School of Mining Engineering in Mons, graduated in mathematics at the University of Liège (1911), with already 67 published papers.

He spends the three following years in Bologna, Göttingen and Paris before rejoining the Belgian army at the beginning of First World War. Sub-lieutenant at the end of the war, he teaches in various military schools, being professor at the Ecole Royale Militaire in 1920. He becomes professor at the Université de Liège in 1925, where he teaches geometry and analysis till his retirement in 1958. As founding member of the Société Mathématique de Belgique, Chairman of the Centre Belge de Recherches Mathématiques (CBRM) and of the National Committee of Mathematics, he has had a deep influence of the mathematical life in Belgium. Godeaux has published some 1200 articles and a dozen of books on algebraic, projective and differential geometry, and on the history of mathematics. Elected corresponding fellow (7.6.1930) and fellow (3.6.39) at the Académie royale de Belgique, he was also elected to several foreign academies, and received important awards inside and outside Belgium.



## Acknowledgments

We first proceed to record gratefully the services rendered by the individual commentators, and above all for their great patience in waiting for the appearance of these volumes. We thank Charles-Armand DE LA VALLÉE POUSSIN and Professor Pietro NASTASI for their lively contributions to the life of Charles-Jean DE LA VALLÉE POUSSIN (in short VP), and his connections to Italy. We are grateful to Professor Tom APOSTOL for his commentary on the papers on number theory. We thank Professors Andy BRUCKNER and Brian THOMSON for their comments on de La Vallée Poussin's decomposition theorem and Professor Hans GÜNZLER for comments on the two papers on almost periodic and quasi-analytic functions. We thank professor Grant GUSTAFSON pour his review of the papers on linear differential equations. We are indebted to Professor Ralph HENSTOCK for his analysis of de La Vallée Poussin's contributions to integration theory, to Professor James JENKINS for describing the contributions to conformal representation and to Professor Jean-Pierre KAHANE for his commentary on the papers on Fourier series and related topics. We are grateful to Professor Jean MEINGUET for commenting de La Vallée Poussin's work on min-max problems, and to Professor Gottfried ANGER and the late Professor Antonie MONNA for their introductions to de la Vallée Poussin's work on potential theory. We thank Professor Rolf NESSEL for contributing to the comments on the papers in approximation theory, and Professor Eugene SENETA for his commentary on the contributions in probability theory. We finally are indebted to Professor Rolf STENS for his collaboration in analyzing the paper on interpolation and sampling theory.

It may be in order to say a few words about the publication problems of these *Collected Works*. Originally the Bibliographisches Institut, Mannheim, had agreed to publish them simultaneously with the *Selected Works* of William Henry YOUNG; it was intended to be the first venture of this renowned publisher in putting collected or selected works of eminent mathematical scholars of the past at the ready disposal of the public. It was Srishti CHATTERJI (Lausanne) and Heinrich WEFELSCHIED (Duisburg), the potential editors of the Young volumes, who had, to our great pleasure, made out the deal. Then in 1995 the Bibliographisches Institut decided to give up publishing mathematical works as a whole, and since their aftercomer, the Spektrum Akademischer Verlag, Stuttgart, had no desire to publish collected works, the editors of both large

projects were stranded.

The problem now was to find a new publishing firm. In the Spring of 1993, one of the editors, Paul BUTZER, gave colloquium talks at the University of Palermo, one being a 90-minute lecture on the life and work of VP. The mathematical historian Pietro NASTASI and the many historically interested mathematicians there, including Gianluca VINTI, who spent a year at the time in Palermo, together with their unbelievably efficient archives, left such excellent impressions upon this editor that he asked Professor Nastasi to write up an article on VP's connections with the *Circolo Matematico di Palermo*. In fact, VP became a fellow of the *Circolo* in 1907, an editorial board member of the famous *Rendiconti* in 1909, and published his basic paper on Fourier series there in 1911.

At the conference on Functional Analysis and Approximation Theory, held at Maratea in September 1996, this editor spoke with its chairman Francesco ALTOMARE about the problem of finding a publisher for the collected works. Professor Altomare suggested the *Circolo* at Palermo which would publish his Proceedings, and promised to contact Professor Pasquale VETRO, Vice-President of the *Circolo* and an editor of the *Rendiconti*, in the matter. Within a short time both parties happily agreed that the Collected Works be published by the *Circolo Matematico di Palermo* as a joint venture with the *Académie royale de Belgique*. The two chief editors express their special thanks to Baron Philippe ROBERTS-JONES, Permanent Secretary of the *Académie Royale de Belgique*, for his enthusiastic and enduring help in establishing the exchange agreement with the *Circolo*, and to the Class of Sciences of the *Académie* for providing the necessary local financial support to the co-edition. This collaboration of *Circolo Matematico di Palermo* and the *Académie Royale de Belgique* is especially appropriate, as VP was associated not only with both academic circles, but was also a foreign member of three other academies in Italy and the Vatican.

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